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MASS TRANSFER IN A CONICAL CHANNEL IN  
THE PRESENCE OF PROCESSES OF EVAPORATION  
AND CONDENSATION AT THE CHANNEL WALLS

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The transfer of the wall material of a channel, caused by the presence of a temperature gradient along the channel axis, is discussed.

In the presence of a temperature gradient along the  $z$  axis of a channel the transfer of matter evaporating from the walls takes place from regions with high temperatures to regions with lower temperatures. If the temperature level is not very high, the vapor flow takes place in the free-molecule mode. The problem of the mass transfer in a cylindrical channel in the presence of a constant temperature gradient along the channel axis was solved in [1]. But the transfer of matter evaporating from the walls can lead to a change in the channel geometry, so that it is interesting to investigate mass transfer in channels with a more complicated geometry. The mass transfer in a conical channel is studied in the present paper. With slow variation of the radius of a real channel along the  $z$  axis the shape of the channel in the vicinity of any point can be approximated by a cone. Thus, knowing the solution for a conical channel, one can approximately calculate the mass transfer in a channel of more complicated shape.

We will take the coefficient of condensation as equal to unity and the velocity distribution of the evaporating molecules as Maxwellian at the wall temperature. In this case the geometrical quantities determining the vapor flow coincide with the corresponding quantities for a noncondensing gas with reflection of a diffuse character. Using the values calculated for them in [2-4], we can write the expression for the mass flux passing through the channel cross section at the point  $z$ . In doing this we will assume that a temperature  $T_0$  and the vapor saturation pressure  $P_0$  corresponding to it are maintained at the channel entrance (at  $z=0$ ), while at the other end the channel opens into a vacuum:

$$G = \frac{1}{2} \sqrt{\frac{\pi m}{2k}} \frac{P_0}{\sqrt{T_0}} \left[ \frac{z^2}{\cos^2\theta} + 2r_0^2 + 2r_0 r' z - \frac{z}{\cos^2\theta} \sqrt{z^2 + 4(r_0^2 + r_0 r' z) \cos^2\theta} \right] + \sqrt{\frac{\pi m}{2k}} \int_0^L \frac{P(z')}{\sqrt{T(z')}} \left[ \frac{z' - z}{\cos^2\theta} + r_z r' - \text{sign}(z' - z) \frac{1}{\cos^2\theta} \frac{(z' - z)^2 + 2r_z^2 \cos^2\theta + 3r_z r' (z' - z) \cos^2\theta}{\sqrt{(z' - z)^2 + 4[r_z^2 + r_z r' (z' - z)] \cos^2\theta}} \right] dz', \quad (1)$$

where  $T(z)$  is the temperature of the channel walls in degrees Kelvin;  $P(z)$  is the vapor saturation pressure at the temperature  $T(z)$ .

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To find the radial mass flux density at the channel wall one can use the continuity equation

$$q_R = -\frac{\cos \theta}{2\pi r_z} \frac{dG}{dz}. \quad (2)$$

Let us consider (1) at  $r' \ll 1$ . In this case one can perform an expansion by powers of  $r'$ ; the first term of the expansion is written in the form

$$G(z) = \sqrt{\frac{\pi m}{2k}} \left[ \frac{1}{2} \frac{P_0}{\sqrt{T_0}} (z^2 + 2r_0 r_z - z\sqrt{z^2 + 4r_0 r_z}) + \int_0^L \frac{P(z')}{\sqrt{T(z')}} \left( (z' - z) - \text{sign}(z' - z) \frac{(z' - z)^2 + 2r_z^2}{\sqrt{(z' - z)^2 + 4r_z^2}} \right) dz' \right]. \quad (3)$$

We note that Eq. (3) with  $r_z \equiv r_0$  describes the mass transfer in a cylindrical channel. In the case of small temperature gradients the integrands in (1) and (3) comprise the product of a slowly varying function  $P/\sqrt{T}$  and a function declining as  $(z' - z)^{-3}$  at  $|z' - z| \gg r_z$ . Thus, a narrow region near the point  $z$  makes the main contribution to the integral. For an approximate calculation of the integrals we expand  $P(z')/\sqrt{T(z')}$  in the vicinity of the point  $z$  and confine ourselves to the linear term of the expansion. Having performed the calculations, we obtain

$$G(z) = \frac{1}{2} \sqrt{\frac{\pi m}{2k}} \left[ \frac{P_0}{\sqrt{T_0}} (z^2 - 2r_0 r_z - z\sqrt{z^2 + 4r_0 r_z}) + \frac{P(z)}{\sqrt{T(z)}} (\psi(z) - \psi(L - z)) + \frac{d}{dz} \frac{P(z)}{\sqrt{T(z)}} \left( -z\psi(z) - (L - z)\psi(L - z) + \chi(z) + \chi(L - z) - \frac{16}{3} r_z^3 \right) \right], \quad (4)$$

where  $\psi(z) = z\sqrt{z^2 + 4r_z^2} - z^2$  and  $\chi(z) = \frac{1}{3} [(z^2 + 4r_z^2)^{3/2} - z^3]$ . Far from the ends of the channel [ $z \gg r_z$  and  $(L - z) \gg r_z$ ] (4) changes into

$$G = -\frac{8}{3} r_z^3 \sqrt{\frac{\pi m}{2k}} \frac{d}{dz} \frac{P}{\sqrt{T}}. \quad (5)$$

This expression coincides in form with the expression describing free-molecule gas flow in a cylindrical channel, but the gradient of saturation pressure and the channel radius at the point under consideration figure in it.

At the ends of the channel (4) is written in the form

$$G(0) = \sqrt{\frac{\pi m}{2k}} \left[ r_0^2 \left( \frac{P_0}{\sqrt{T_0}} - \frac{P(0)}{\sqrt{T(0)}} \right) - \frac{4}{3} r_0^3 \frac{d}{dz} \frac{P}{\sqrt{T}} \Big|_{z=0} \right], \quad (6)$$

$$G(L) = \sqrt{\frac{\pi m}{2k}} \left[ r_L^2 \frac{P(L)}{\sqrt{T(L)}} - \frac{4}{3} r_L^3 \frac{d}{dz} \frac{P}{\sqrt{T}} \Big|_{z=L} \right]. \quad (7)$$

From (5)-(7) it is seen that far from the ends of the channel the mass transfer is connected with the presence of a temperature gradient, while at the ends of the channel the mass flow is determined both by the temperature gradient and by the difference in the values of  $P/\sqrt{T}$  at the end of the channel and in the void into which the channel emerges. Thus, with isothermal heating the mass loss occurs mainly from the ends of the channel, while in the presence of a temperature gradient there is also appreciable mass loss from the interior of the channel.

#### NOTATION

$m$ , mass of a molecule;  $\theta$ , aperture angle of cone;  $r_0$ , channel radius at  $z=0$ ;  $r_z$ , current value of channel radius;  $r' = dr_z/dz$ ;  $L$ , channel length;  $k$ , Boltzmann's constant.

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